# 21-268 Final review questions

1. Let $C = \left\{ (x,y) \in \mathbb{E}^2 \,/\, \begin{matrix} 0<x<1 \\ 0<y<1 \end{matrix} \right\}$. Show $C$ is open.

2. Define $f(x,y) = \begin{cases} \dfrac{|y|^{9/2}}{x^4+y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

   At what pts do $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist? Fail to exist?

3. Find the following limits as $(x,y) \to (0,0)$ either by showing they do not exist or providing a short but precise justification.

   a) $\dfrac{x^2 y^3}{x^6 + y^4}$ b) $\dfrac{x^4 y}{x^8 + y^2}$.

4. Find the total differential (ie $a, b, \varepsilon_1, \varepsilon_2$) of $f(x,y) = xe^y$ at $(0,0)$.

5. Consider solving $\begin{cases} xy + z^2w^2 - 1 = 0 \\ xz + y^2w^2 - 1 = 0 \end{cases}$ For $(w,x)$ in $(y,z)$

   near $(w,x,y,z) = (1,0,-1,1)$. Does the implicit ft theorem apply?

   Find a system of lin. eqn. that $\frac{\partial w}{\partial y}, \frac{\partial x}{\partial y}$ have to solve there.

(6) let $z = f(x,y)$ with $f \in \mathcal{C}^2$.
let $x = r^2 + t^2$, $y = rt^2$.

Find $\dfrac{\partial^2 z}{\partial r \partial t}$ (in terms of $r, t, f_x, f_y, f_{xx}$ etc...)

(7) Let $f(x,y) = xy^2$ and $D = \left\{ (x,y) \,/\, x^2+y^2 \leq 4 \text{ and } x \geq 0 \right\}$

a) why Does $f$ have an absolute min./max on $D$?

b) Find the points where they are reached and their values.

(8) let $f(x,y) = (x^2-4)^2 + (y^2-4)^2$ on $E^2$.

Find all critical pts of $f$, determine which one are loc. min / max / saddle points.

(9) let $\vec{g}(x,y,z) = (2xy^3z^4 + 5x^4z)\vec{i} + (ze^{yz} + 3x^2y^2z^4)\vec{j} + (x^5 + ye^{yz} + 4x^2y^3z^3)\vec{k}$

a) Compute $\vec{\nabla} \times \vec{g}$.

b) Find $f(x,y,z)$ such that $\vec{g} = \vec{\nabla} f$.

(10) let $f(x) = 5x+3$ on $[a,b] = [1,4]$. let $x_0 = 1 < x_1 < \ldots < x_n = 4$,
$h = \max\limits_{i=1\ldots n} (x_i - x_{i-1})$.

a) What common limit of Rieman sum do you expect to find?

b) let $m_i = \frac{x_i + x_{i-1}}{2}$ ; show that $\sum_{i=1}^{n} f(m_i)(x_i - x_{i-1}) = \frac{93}{2}$.

c) let $x_i^* \in [x_{i-1}, x_i]$ for $i = 1 \dots n$. Show that

$$\left| \sum_{i=1}^{n} f(x_i^*)(x_i - x_{i-1}) - \frac{93}{2} \right| \xrightarrow[h \to 0]{} 0 .$$