21-268 – Homework assignment week #2

Laurent Dietrich
Carnegie Mellon University, Spring 2017, Sec. A and B

Reminder

Homework will be given on Wednesdays and due on the next Wednesday before 5pm, to me in class or in Christopher Cox's mailbox in Wean Hall 6113 (pay attention to the arrow !). Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me your TA can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Reading

1. Proofs in the textbook that I did not do in class.

Exercises (29 pts)

1. (3 pts) Let

$$f(x,y) = \frac{xy^3}{x^4 + y^4 + x^2 + 0.01y^2}.$$

    For any $\epsilon > 0$ find $\delta > 0$ such that $0 < \sqrt{x^2 + y^2} < \delta$ implies $|f(x,y)| < \epsilon$. Conclude.

2. (4*2 pts) For each of the following functions determine if $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$ exist and find them if they do:

    (a)
    $$f(x,y) = \begin{cases} \dfrac{x^2}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

    (b)
    $$f(x,y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

    (c)
    $$f(x,y) = \begin{cases} \dfrac{|y|^3}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

    (d)
    $$f(x,y) = \begin{cases} \dfrac{|y|^{3.1}}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

3. (2*2 pts) Below are pictures of some level curves of four functions.

    (a) For only one of the four we have $\frac{\partial f}{\partial x}(1,2) = 3$. Which one and why ?

    (b) For only one of the four we have $\frac{\partial f}{\partial y}(3,2) = -1$. Which one and why ?
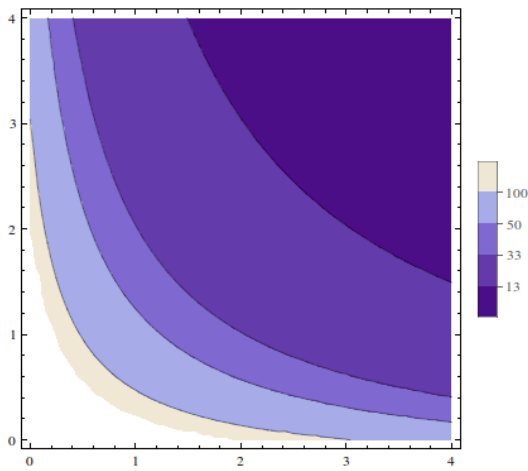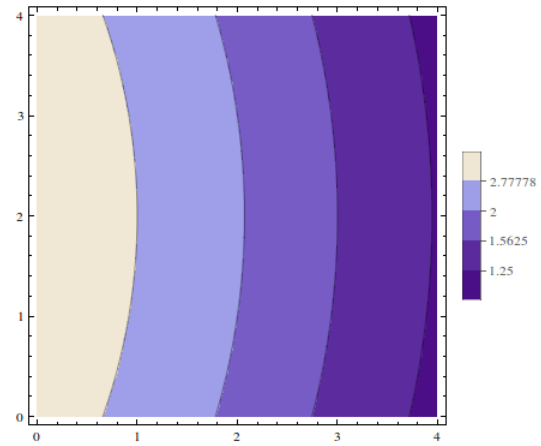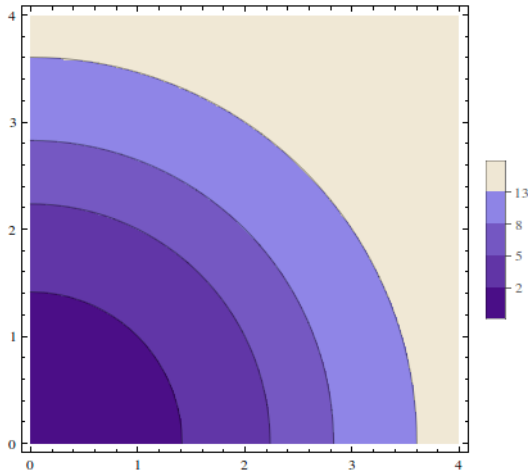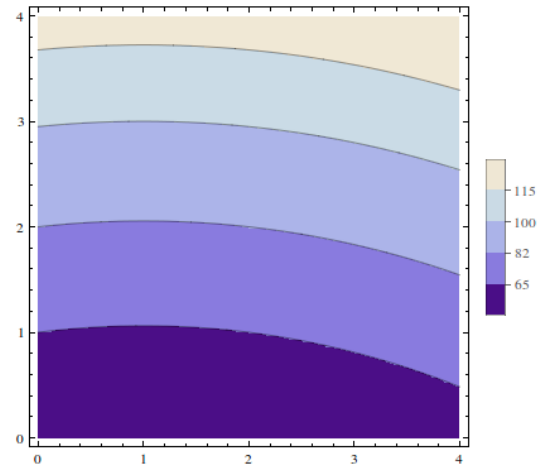
Figure 1:

Figure 2:

Figure 3:

Figure 4:

4. (4+2 pts) Let

$$f(x, y) = \begin{cases} \dfrac{x^3 y}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Find $\frac{\partial f}{\partial x}(x, y)$ and $\frac{\partial f}{\partial y}(x, y)$ at all points (x, y), including $(0, 0)$. Then find $\frac{\partial^2 f}{\partial y \partial x}(0, 0)$ and $\frac{\partial^2 f}{\partial x \partial y}(0, 0)$.

5. (2+1 pts) Evaluate $\dfrac{\partial z}{\partial x}(x, y)$ and $\dfrac{\partial z}{\partial y}(x, y)$ if $xy^2 + yz^3 + xyz = 1$.

6. (2+2+1 pts) Consider the set of equations in $\mathbb{E}^5$

$$\begin{cases} uvw^2 xy^3 = 1 \\ u^2 v^3 wx^2 y = 1 \end{cases}$$

Find $\dfrac{\partial u}{\partial x}(x, v, w)$ (v, w being held constant) on the one hand by implicit differentiation and on the other hand by solving the equations for u and y in terms of x, v, and w. Show that results are the same.

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