

21-241 – Homework assignment week #4

Laurent Dietrich
Carnegie Mellon University, Fall 2016, Sec. F and G

Reminder

Homework will be given on Fridays and due on the next Friday before 5pm, to me in class or in Andrew Zucker's mailbox in Wean Hall 6113. Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me and Andrew Zucker can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Reading

- Poole, Sec. 6.4.

Exercises

1. Let a, b be any real numbers and

$$(S) \begin{cases} x + y = a \\ 2x - 3y = b \end{cases}$$

Solve (S) . Rewrite (S) using the column-by-column approach. Interpret the results. What is the interpretation from the row-by-row point of view ?

2. In the following, determine whether W is a subspace of V .

i) $V = \mathbb{R}^3, W = \left\{ \begin{bmatrix} x \\ y \\ x + y + 1 \end{bmatrix} \mid x, y \in \mathbb{R} \right\}$

ii) $V = \mathcal{M}_{nn}(\mathbb{R}), W$ is the subset of *diagonal matrices*, that is, matrices with non-zero entries only on the diagonal (the one from top-left to bottom-right).

3. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be linear and such that

$$T \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \text{ and } T \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

Can you compute $T \begin{bmatrix} a \\ b \end{bmatrix}$ for any $a, b \in \mathbb{R}$ (justify your answer) ? If yes, do so.

4. Find two matrices A and B such that $(A + B)^2 \neq A^2 + 2AB + B^2$. When is this relation actually satisfied ?
5. Prove Theorem 1)v) from Chapter 5. That is, if A is any $m \times n$ matrix then

$$I_m A = A = A I_n$$