# 21-241 – Homework assignment week #14

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**Reminder**

Homework will be given on Fridays and due on the next Friday before 5pm, to me in class or in Andrew Zucker's mailbox in Wean Hall 6113. Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me and Andrew Zucker can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

## Reading

1. Poole: Sec. 5.4 and 6.6.

## Exercises (19 pts)

### Coordinates and change of basis (2*4 pts)

Find the coordinates $[x]_B$ and $[x]_C$ in the following situations by solving a linear system. Compute the change-of-basis matrix $P_{C \leftarrow B}$. Compute $[x]_C$ thanks to this matrix and make sure that the result agrees with the first one.

1. $x = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $B$ the standard basis of $\mathbb{R}^3$, $C = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.
2. $x = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, $B = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$, $C = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$.

### Matrices of orthogonal projections (3*2 pts)

Let $W$ be in $\mathbb{R}^3$ the plane with equation $x - y + 2z = 0$.

1. Show that $p : \mathbb{R}^3 \to W$ the orthogonal projection onto $W$ is a linear transformation.
2. If $(v_1, v_2)$ is an orthogonal basis of $W$ and $v_3$ a unit vector of $W^\perp$, and $C = (v_1, v_2, v_3)$, what is $[p]_C$ ?
3. Compute the matrix of $p$ from the standard basis to the standard basis (we call it the *standard matrix*).

### Orthogonal diagonalization of a symmetric matrix (5 pts)

Let

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Find $Q$ orthogonal and $D$ diagonal such that $Q^T A Q = D$.